Department of Agricultural Economics  
PhD Qualifier Examination  
January 2006

Instructions:

The exam consists of seven questions. You must answer all questions. If you need an assumption to complete a question, state the assumption clearly and proceed. Be as clear in your answer as possible. You have four hours to complete the exam. Be sure to put your assigned letter and no other identifying information on each page of your answer sheets. Also, put the question number and answer page number at the top of each page. Finally, please write on only one side of your paper and the leave the appropriate margins.

Good Luck!
(15 points)

1. An economic agent’s preferences are characterized by the indirect utility function
   \[ V(p, m) = p_1^{-1/4} p_2^{-2/4} m^{3/4}. \]

   (a) Are the underlying preferences homothetic? Justify your answer.

   (b) Find the agent’s Marshallian and Hicksian demand functions.

   (c) Compute the Slutsky derivatives, \( S_{11}(p,m) \) and \( S_{22}(p,m) \).

   (d) Find the agent’s expenditure function.

   (e) How would you classify the goods in terms of income (e.g., inferior, normal)?
       Does your answer change if the indirect utility function is of the general form \( V(p, m) = p_1^{-a} p_2^{-b} m^{c} \)? Explain.
(10 points)

2. Consider a firm with one output, q, and two inputs, K and L. The prices for the two inputs are v and w, respectively. The cost function for the firm is assumed to be of the following functional form:

$$\ln C(q, v, w) = \ln q + \beta_0 + \beta_1 \ln v + \beta_2 \ln w + \beta_3 (\ln v)^2 + \beta_4 (\ln w)^2$$
$$+ \beta_5 \ln v \ln w$$

a) What is the assumption concerning returns to scale implied here?

b) What parameter restrictions are implied by homogeneity of the cost function?

c) Show that input shares are linear in the parameters.

d) Derive an expression for the partial elasticity of substitution between capital and labor in terms of parameters and input shares.

e) Under what parameter restrictions is this cost function of the Cobb-Douglas form?
(20 points)

3. A basic result of economic theory is that the price of an input is determined to some extent by the returns (profit) generated from that input. This implies that the price of agricultural land should be determined by the returns from using the land in a particular activity. Furthermore, the factors that affect those returns, such as crop subsidies or land quality, should affect the price of agricultural land.

In a recent article in the Review of Agricultural Economics, entitled “Noncash Income Transfers and Agricultural Land Values”, Taylor and Brester investigated the impact of sugar beet prices on agricultural land prices in Montana. Below are the variable definitions, means, and regression results.

<table>
<thead>
<tr>
<th>Variable</th>
<th>Definition</th>
<th>Mean</th>
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<tbody>
<tr>
<td>P</td>
<td>Deflated price per acre of parcel of land sold.</td>
<td>$697.50</td>
</tr>
<tr>
<td>R</td>
<td>Deflated expected price of sugar beets per ton in Montana.</td>
<td>$29.10</td>
</tr>
<tr>
<td>Q</td>
<td>An index of soil quality of parcel.</td>
<td>.65</td>
</tr>
<tr>
<td>QR = R×Q</td>
<td>Quality adjusted deflated price per ton of sugar beets. Calculated as expected price of sugar beets per ton (R) × index of soil quality of parcel (Q).</td>
<td>$29.10</td>
</tr>
<tr>
<td>C</td>
<td>Deflated annual per acre expected cash receipts from crop sales (in Montana).</td>
<td>$87.59</td>
</tr>
<tr>
<td>S</td>
<td>Size of Parcel sold, in acres.</td>
<td>282.61</td>
</tr>
</tbody>
</table>

The OLS results are

\[ \hat{P}_i = 635.89 + 3.85QR_i + .76C_i - .62S_i + .0001S_i^2 \]

\[ (.00) \quad (.08) \quad (.07) \quad (.00) \quad (.00) \]

\[ R^2 = .28 \]

Cook-Weisberg Heteroskedasticity test (p-value for null of no heteroskedasticity) = 1.00. Sample size (parcels sold) = 550.

The subscript denotes the parcel of land and the numbers in parentheses are p-values corresponding to the parameter estimates.

Based on this information, answer the following questions.

a. Give an economic interpretation of the parameter estimates and indicate if they agree with theory.
b. Give a statistical interpretation of the results and indicate if there are any estimation problems that need to be corrected. If there are any problems that need to be corrected, explain how you would correct them.

c. Assuming the estimation is acceptable, calculate the elasticity of the price of land \((P)\) with respect to quality adjusted price \((QR)\) and size \((S)\) evaluated at the mean values and indicate which variable seems to be most important.

d. The model above implicitly imposes a restriction on the elasticity of the price of land with respect to the expected price of beet \((R)\) and the elasticity of the price of land with respect to the quality of soil \((Q)\). Show what this restriction is and describe how you could test if this is an acceptable restriction using a linear functional form (not a loglinear functional form).
(10 points)

4. The concept of mean square error (MSE) is a very important concept in statistics and econometrics as it brings together two important concepts: efficiency and bias.

a. Define efficiency and bias.

b. Define MSE.

c. Using a graph of the sampling distributions for two estimators of the same parameter and the concept of MSE, explain why a biased estimator may be preferred to an unbiased estimator. In your answer be sure to make use of the distinction between the estimator and the estimate.
(10 points)

5. The theory of revealed preference allows prediction of the consumer’s behavior without specification of an explicit utility function, provided the consumer conforms to some simple axioms.

Assume two commodities, q₁ and q₂. At the initial prices, implicit in the budget line \( p^0 \), the consumer purchases commodity combination \( q^0 \). When facing prices \( p^1 \), the consumer purchased \( q^1 \). Two different scenarios, a and b are given graphically below.

![Graph](image)

a) State the weak axiom of revealed preference.

b) In which scenario, can the consumer purchase \( q^1 \) at the \( p^0 \) prices?

c) Which scenario violates the weak axiom of revealed preferences and why?
(15 points)

6. Ellsberg studied the problem of an agent who has to choose between gambles that involve two urns. In both urns there are red, blue, and green balls, 90 total balls in each.

In Urn 1 there are 30 balls of each color.

In Urn 2 there are 30 red balls and 60 balls that are blue and green, but the distribution between blue and green is not known.

The two gambles yield a payoff of $3 if a green ball is chosen.

   a. Assuming that the individual is risk neutral, derive the maximum amount that he or she would be willing to pay to participate in the lottery involving Urn 1.

   b. Show that if the agent’s utility function is concave, he or she would be willing to pay less to participate in the lottery than the amount found in part a. How is this related to risk aversion?

   c. Consider a lottery involving Urn 2. Assume the individual’s believes that the probability that a green ball is drawn, say $p_g$, is uniformly distributed between 0 and \( \frac{2}{3} \); i.e. $p_g \sim \mathcal{U}\left[0, \frac{2}{3}\right]$. If the individual’s utility function is $u(x) = x^\gamma$, with $0 < \gamma < 1$, derive the maximum amount that he or she would be willing to pay to participate in the lottery.
(20 points)

7. Allison and Betty are farmers. The Department of the Environment (DOE) is willing to pay them to install a pollution reducing pond on their land. However, the state cannot actually observe the costs of the farmers, it must rely upon the costs reported by the two farmers. The farmers report an integer cost to the DOE in a sealed bid. The agency knows that the cost is between $40 and $100 and bids must fall in this range. Allison and Betty are both dishonest and will get as much money from the state as they can, they don’t care about the other farmer’s payoff.

To discourage cheating, the agency will pay both farmers the same price, equal to the lowest reported cost.

The farmers both know that the true cost to build the pond is $50.

a. What will be the equilibrium bids of the two farmers?

b. Now suppose that the agency imposes a fine of $5 on a farmer if her bid is higher than the other farmer’s bid. How would this change the equilibrium?

c. Suppose that the agency changes the rules so that the farmer who makes a higher bid must make a $5 side payment to the other farmer. Both farmers are still paid the low bid by the agency if they build the pond. How would this change the equilibrium?

d. Now suppose that the program is set up as in c, but that after all bids have been placed and the results have been announced, the farmers can drop out of the program. If a farmer drops out she will not incur any costs and all side payments are cancelled. How does this affect the equilibrium?