Instructions:

The exam consists of six questions. You must answer all questions. You have four hours to complete the exam.

If you need an assumption to complete an answer, state the assumption clearly and proceed with your answer. Label each answer page you produce with your assigned letter identification. Do not put any other identifying information on the answer pages. In addition place the question number on each sheet (page) of paper. Write on just one side of the page and leave sufficient margins for copying and grading comments.
1. (20 points) Given the following data to be analyzed under the maintained hypothesis of utility maximization:

\[ x_1^0 = 100, \quad p_1^0 = 2, \quad \frac{\partial x_1}{\partial p_1} = -50, \text{and} \quad \frac{\partial x_1}{\partial w} = 0.05 \]

Where \( x_1^0 \) is the quantity of good 1 consumed in the initial period, \( p_1^0 \) is the price of good 1 in the initial period and \( w \) is wealth.

Suppose the price of good 1 increases to \( p_1^1 = 3 \). How much should a public assistance program aimed at maintaining a certain standard of living be increased to offset this price increase?

a) Explain the relevance of the concept of Compensating Variation (CV) to this question.

b) Use the Slutsky equation to derive a linear approximation to the CV for this problem.

c) What is the relationship between your CV approximation measure and a Marshallian consumer surplus approximation measure of the required compensation?

d) Discuss how you would extend this welfare analysis to the case where a second price, say \( p_2 \), also changes.
2. (15 points) Given the production function,

\[ F(L, K) = \sqrt{L} + \sqrt{K}, \]

Is the underlying technology convex? Discuss returns to scale and whether factors exhibit diminishing returns. For this technology, is the ability to substitute between labor and capital easier or more difficult than one characterized by a Cobb-Douglas production function? In answering the above, include formal definitions for each of the underlying economic concepts.

Next, discuss the relation between conditional demands and derived demands, then illustrate explicitly for this technology.

Finally, when making comparisons between these two demands, identify which one would exhibit a greater own-price elasticity of demand and explain why this would necessarily be expected.
3. Suppose \( Y_i = \beta + \alpha X_i + \varepsilon_i \), where \( \varepsilon_i \) is an independently and identically distribution error term with mean zero and finite variance \( \sigma^2 \).
   a. Derive the mean and variance of \( Y_i \). (5 points)

   b. Derive the conditional mean and variance of \( Y_i \) given \( X_i \), assuming that 
   \[ E(\varepsilon_i | X_1, \ldots, X_n) = 0 \text{ for } i = 1, 2, \ldots, n. \] (5 points)

   c. Show that the OLS estimator is biased if 
   \[ E(\varepsilon_i | X_1, \ldots, X_n) \neq 0. \] (5 points)

   d. Outline a solution to the problem described in part c. State clearly if any additional information and/or assumptions are required to warrant a consistent estimator. (5 points)
4. Consider the panel data model \( Y_{it} = X_{it} \beta + c_i + u_{it}, i=1, \ldots, n, t=1, \ldots, T \), where \( c_i \) is the individual effect that is time invariant. The error term \( u_{it} \) is strictly exogenous in the sense that \( E(u_{it}|X_{it}, \ldots, X_{iT}, c_i) = 0 \) for all \( i \).
   
a. Explain whether the simple (pooled) OLS estimator is consistent. (5 points)

b. If we further assume that \( E(c_i|X_{it}, \ldots, X_{iT}) = 0 \), outline an estimator that explicitly takes into account the panel structure of the model. Explain how to implement the proposed estimator. (10 points)
5. A game between duopolists, firm A and firm B, has the following hypothetical payoff matrix.

<table>
<thead>
<tr>
<th>Firm B</th>
<th>( Q_A = 128 )</th>
<th>( Q_A = 96 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( Q_B = 128 )</td>
<td>$16.4 \backslash $16.4</td>
<td>$20.5 \backslash $15.3</td>
</tr>
<tr>
<td>( Q_B = 96 )</td>
<td>$15.3 \backslash $20.5</td>
<td>$18.4 \backslash $18.4</td>
</tr>
</tbody>
</table>

Here the matrix illustrates that A and B can both supply two levels of product, 128 or 64 thousand units per year (\( Q_B = 128 \) or \( Q_B = 96 \) and \( Q_A = 128 \) or \( Q_A = 96 \)). The payoffs to each firm are given in the body of the table in millions of dollars. Within each cell there are two values listed along with a “\( \backslash \)” sign. The first listed payoff in each cell is that going to Firm B; the second listed payoff within a cell is that going to Firm A. So for example, if Firm A supplies 96 thousand units and firm B supplies 128 thousand units, the payoff to firm A is $15.3 million per year and the payoff to firm B is $20.5 million per year.

a. Find the non-cooperative Nash equilibrium (5 points).

b. If the market demand for this product is given as: \( Q = 678 - P \); where \( Q = Q_A + Q_B \) and \( P \) is the dollars price of the product. For constant marginal cost for both firms of $294 per unit, calculate firm A’s best response given B’s response and B’s best response given A’s response. Use these response functions to calculate the Cournot equilibrium (10 points).
6. (15 points) Assume that yields of a certain crop follow the discrete distribution presented in the table and illustrated in the figure:

<table>
<thead>
<tr>
<th>Yield</th>
<th>Probability</th>
</tr>
</thead>
<tbody>
<tr>
<td>70</td>
<td>1/6</td>
</tr>
<tr>
<td>90</td>
<td>1/4</td>
</tr>
<tr>
<td>110</td>
<td>1/2</td>
</tr>
<tr>
<td>130</td>
<td>1/12</td>
</tr>
</tbody>
</table>

A typical yield insurance contract with a coverage level \( \eta \)\% of the expected yield \( \bar{y} \) pays an indemnity of \( I = \bar{y} \eta - y \) whenever the actual realized yield \( y \) falls below the protected level \( \bar{y} \eta \).

(a) An insurance company CBA offers two yield insurance contracts with coverage levels of 100% and 80% of the expected yield. Calculate the expected yield \( \bar{y} \) and the actuarially-fair premiums for the two contracts based on the distribution of yields above. Normalize the price of the crop to 1.

(b) Farmer Andrew has a utility function \( u_A = \sqrt{R} \), while farmer Betty has a utility function \( u_B = 0.5R \), where \( R \) is the total revenue from selling the crop plus a (potential) insurance payoff less insurance premium. Using your results from part (a) and assuming that both Andrew and Betty are expected utility maximizers, determine which contract (if any) each farmer will buy if the contract prices are actuarially fair. Once again, normalize the crop price to 1.

(c) Insurance companies often adjust insurance premiums up from the actuarially-fair level in order to account for unexpected losses and additional expenses. The adjustment usually consists in multiplying the actuarially-fair premiums by \( 1 + \gamma \), where \( \gamma \) is a loading factor (or simply a load). Assuming that Andrew and Betty are offered premiums with a 10% load, calculate the optimal contract choices for each farmer. Keep in mind that a farmer might be better off without a contract.